On the mutual polarization of atoms He^4 and a possible nature of the electric activity of He II

V. M. Loktev and M. D. Tomchenko

Bogoliubov Institute for Theoretical Physics
14-b, Metrologichna Str., Kiev 030143, Ukraine
E-mail:vloktev@bitp.kiev.ua; mtomchenko@bitp.kiev.ua

We propose a simple method based on the standard quantum-mechanical perturbation theory to calculate the mutual polarization of two atoms He^4. A possibility of the explanation of the electric signals observed in several experiments with helium-II is discussed.
PACS: 67.40.Pm, 32.10.Dk
Key words: helium-4, electrical activity, dipole moment, atom polarization.

1   Introduction

As usual, the superfluid He^4 draws attention in view of its superfluid (SF) properties. However, the experiments [1]–[3] revealed electric properties of He II in the cases where external electromagnetic fields are absent. These results were rather unexpected in that the free He^4 atoms possess no charge and dipole or higher multipole moments.

Let us dwell in more details on experiment [1], where a second-sound standing half-wave was generated in He II placed in a metal resonator. The electric signal (potential difference) oscillating with the frequency of the second sound and the amplitude $\triangle U \approx k_B \triangle T/2e$ was observed between the electrode on the resonator wall and ground. Here, e is the electron charge, and $\triangle T$ is the amplitude of temperature oscillations in the second-sound wave.

Although the attempts to understand a nature of this signal were made in several papers [4]–[7], its satisfactory model has not been constructed yet. In particular, it was assumed in [4] that the quadrupole moment generates a dipole moment (DM) of atoms He II in some way. However, neither a mechanism of the DM generation nor a nature of the quadrupole moment were explained. In work [5], an idea was proposed that atoms He II acquire a DM due to their acceleration in the second-sound wave. However, the amplitude $\triangle U$ obtained in such an approach turned out to be two or three orders of magnitude smaller than the observed value. In [6], it was suggested that the results found in [1] are based on the acceleration-induced polarization of atoms [5] and the presence of a large enough entropy $s_s \approx 2k_B$ (per atom) in the SF component of helium. A possible explanation of the origin of this entropy is given in [7]. However, the initial equations were not quite rigorously deduced in Ref. [6], so that the relation for the second sound

$$u_2 = \frac{s_n + s_{sf}}{m_4}\sqrt{\frac{\rho\rho_s T}{\rho_n C(T)}}$$

for $s_s \approx 2k_B$ and $T = 1 \div 2\,K$ yields the value of u_2 by one or two orders of magnitude larger than the observed one (although, for $s_s = 0$, this relation agrees with the experimental value very well).

In those works, the main mechanism of the polarization of atoms was the inertial mechanism [5]: a polarization due to the inertial forces at the acceleration. However, the

inertial forces (and the gravity force) give, as known, the identical acceleration to bodies with different masses. Therefore, in our opinion, the acceleration by itself cannot be a primary source of the polarization of atoms He (see details in [8]). Works [9, 10] advance the idea of the polarization of atoms due to the interaction with one another ("tidal" mechanism). In this case, the acceleration can give a correction to the polarization, but in the frame of the tidal mechanism. It was assumed in [10] that the polarization of He-II can be related to the ordering in an ensemble of vortex rings. However, the value of the DM of a ring was postulated, and, in addition, it was found that the volume polarization of any nature cannot explain the properties of a signal [1], since a volumetric signal must strongly depend on the resonator sizes, whereas the experimental signal is independent of those sizes. The tidal mechanism ensures [9, 11, 8] the polarization of helium in the presence of the gradient of density or temperature. Such gradients allow a second-sound wave to induce a volume polarization in helium and a weak volumetric signal less intense than the experimental one by 1–2 orders [8]. Works [12] and [13] presented the idea of the relationship of the experimental electric signal to the unidirectional polarization of atoms He^4 located at the electrode surface. This idea yields an approximate microscopic model [14] which explains significantly well the properties of the observed signal.

The application of the tidal mechanism requires to know the mutual polarization of two He^4 atoms. It was determined in [15, 16], where it was also shown that each of two interacting atoms He^4 induces a dipole moment

$$\mathbf{d} = -D_7|e|\frac{a_B^8}{R^7}\mathbf{n}, \quad D_7 = 13.2 \div 18.4 \tag{1}$$

on the neighboring atom. Here, $\mathbf{n} = \mathbf{R}/R$ is the unit vector toward the second atom, and $a_B = \hbar^2/me^2 = 0.529\,\text{Å}$. The calculation in [15, 16] was executed on the basis of special dispersion relations known mainly to experts in atomic physics. Therefore, we propose to solve the problem by a simpler method which does not leave the frame of the ordinary course of quantum mechanics and, in this case, will verify the result (1). Such a calculation is also actual because, in order to fit the experiment [1], it is important to know the tidal DM (1) with maximum accuracy [14].

2 Mutual polarization of two atoms He^4

According to the standard perturbation theory (PT), we will search for the ground-state wave function for two immovable interacting atoms He^4, A and B, as the expansion in all possible states $\Psi_{\tilde{m}}$ of two free atoms A and B:

$$\Psi_0^{AB} = \sum_{\tilde{m}_A \tilde{m}_B} c_{\tilde{m}_A \tilde{m}_B} \Psi_{\tilde{m}_A}^A \Psi_{\tilde{m}_B}^B, \tag{2}$$

where $\tilde{m}_A$ and $\tilde{m}_B$ are the complete collections of quantum numbers (n, l, m) for these atoms. By $\Psi_{\tilde{l}} \equiv |\tilde{l}\rangle$, we denote the set of states with a given l and all possible corresponding m and n ($m = 0, \pm 1, \dots \pm l, \ n = l, \dots, \infty$). In what follows, we will omit sometimes the tilde for simplicity: $\Psi_{\tilde{l}} \equiv \Psi_l$, $|\tilde{l}\rangle \equiv |l\rangle$, $c_{\tilde{0}\tilde{0}} \equiv c_{00}$, *etc.* Since the excited *s*-states ($1sns$) give the zero contribution to a DM (4), we neglect them, so that Ψ_0 means the ground state $1s1s$ of an atom He^4. It is obvious that $c_{ij} = c_{ji}$. According to PT, we have

$$c_{ij} = c_{ij}^{(1)} + c_{ij}^{(2)} + \dots, \tag{3}$$

where the upper index shows the order of PT. The tidal DM of atom A is sought by the general formula

$$\mathbf{d}^A = \int \Psi_0^{*AB} \hat{\mathbf{d}}^A \Psi_0^{AB} d\mathbf{r}_1^A d\mathbf{r}_2^A d\mathbf{r}_1^B d\mathbf{r}_2^B, \tag{4}$$

where $\mathbf{r}_j$ are the coordinates of electrons, and

$$\hat{\mathbf{d}}^A = e(\mathbf{r}_1^A + \mathbf{r}_2^A). \tag{5}$$

The function Ψ_0 describes a parastate. Therefore, the coefficients c_{ij} are nonzero only for parastates Ψ_j. In view of this fact, the spin function is the same for all Ψ_j in (2). Therefore, the spins can be took out in (4). Since the sum over spins gives 1, we omit the spin coordinates.

We note that the quantity $d^A \sim c_{10}(1) \sim (a_B/R)^4$ was determined in [10]. The coefficient c_{10} was found in the first order of PT, but its calculation includes an inaccuracy: in the charge operator $\hat{Q}$ (see formulas (11)–(13) from [10]), the Z-axis was chosen prior to the differentiation. But this axis should be fixed after all differentiations. In this last case, all the coefficients $c_{10}^{(1)}$ become zero, because $c_{10}^{(1)} \sim \langle 0^B | \hat{Q}_B | 0^B \rangle R^{-1}$ and

$$\langle 0^B | \hat{Q}_B | 0^B \rangle R^{-1} = 0. \tag{6}$$

In particular, $\langle 0^B | \hat{Q}_{B,q} | 0^B \rangle R^{-1} \sim \sum_{\alpha\beta} \delta_{\alpha\beta} \frac{\partial}{\partial R_\alpha} \frac{\partial}{\partial R_\beta} \frac{1}{R} = \triangle \frac{1}{R} \equiv 0$, and we get zero only for the Coulomb potential. Relation (6) is valid, because an helium atom has no moment in the ground state.

In this connection, we will determine c_{10} in the second order of PT:

$$c_{10}^{(2)} = -\frac{\langle 00 | \hat{U} | 00 \rangle \langle 10 | \hat{U} | 00 \rangle}{(E_{0,0}^{(0)} - E_{1,0}^{(0)})^2} + \sum_{\tilde{m} \neq (\tilde{0}, \tilde{0})} \frac{\langle \tilde{m} | \hat{U} | 00 \rangle \langle 10 | \hat{U} | \tilde{m} \rangle}{(E_{0,0}^{(0)} - E_{\tilde{m}}^{(0)})(E_{0,0}^{(0)} - E_{1,0}^{(0)})}, \tag{7}$$

where $\tilde{m} = (\tilde{m}_A, \tilde{m}_B)$, and

$$\langle \tilde{m}_1 | \hat{U} | \tilde{m}_2 \rangle = \int \Psi_{\tilde{m}_1^A}^{*A} \Psi_{\tilde{m}_1^B}^{*B} \hat{U} \Psi_{\tilde{m}_2^A}^A \Psi_{\tilde{m}_2^B}^B d\mathbf{r}_1^A d\mathbf{r}_2^A d\mathbf{r}_1^B d\mathbf{r}_2^B. \tag{8}$$

It follows from $\hat{U} = \hat{Q}_A^+ \hat{Q}_B(1/R)$ [17] and (6) that $\langle 00 | \hat{U} | 00 \rangle = 0$. Therefore, only the second term on the right-hand side in (7) is of importance. The main contribution to this term is given by the states $\tilde{m} = (1, 2)$ and $\tilde{m} = (2, 1)$. We have

$$c_{10}^{(2)} = \sum_{\tilde{m} \neq (\tilde{0}, \tilde{0})} \frac{\langle \tilde{m} | \hat{U} | 00 \rangle \langle 10 | \hat{U} | \tilde{m} \rangle}{\triangle E_{\tilde{m}} \triangle E_{1,0}}. \tag{9}$$

As n_1 and n_2 increase, the terms in (9) decrease. The calculation indicates that $\mathbf{d}^A$ receives a significant contribution from a lot of states at c_{10} with $n > 2$ (in particular, $n = 5$ and 6). Since the number of states is huge, it seems impossible to find and to sum over all the required states in (9), as well as all c_{10} with different n and m.

Nevertheless, this difficulty can be avoided if the series is summed approximately. We note that the difference of energies for an atom He^4 is minimum for those of the first excited level $1s2p$ and the ground state, $\triangle E_1(n = 2) = 21.236\,\text{eV}$, and the maximum difference is represented by the ionization energy $\triangle E_{ion} = 24.58\,\text{eV}$ [18]. In (9), a significant contribution is given, for example, by the state $1s5p$ c $\triangle E_1(n = 5) = 24.066\,\text{eV}$ [18]. Since $\triangle E$ are sufficiently close for different n, we set them to be equal to a constant $\triangle E$.

Let us supplement (9) by the term $\tilde{m} = (\tilde{0}, \tilde{0})$ $\langle 00|\hat{U}|00\rangle\langle 10|\hat{U}|00\rangle/2\triangle E^2$ (it is equal to zero due to $\langle 00|\hat{U}|00\rangle = 0$). With regard for the fact that $\triangle E_{1,0} \equiv E_{0,0}^{(0)} - E_{1,0}^{(0)} = 2E_0^{(0)} - E_1^{(0)} - E_0^{(0)} = E_0^{(0)} - E_1^{(0)} \approx -\triangle E$ and $\triangle E_{\tilde{m}} \equiv E_{0,0}^{(0)} - E_{\tilde{m}}^{(0)} = 2E_0^{(0)} - E_{\tilde{m}_A}^{(0)} - E_{\tilde{m}_B}^{(0)} \approx -2\triangle E$ in (9), we obtain

$$c_{10}^{(2)} \approx \frac{1}{2\triangle E^2} \sum_{\tilde{m}} \langle 10|\hat{U}|\tilde{m}\rangle\langle \tilde{m}|\hat{U}|00\rangle = \frac{1}{2\triangle E^2}\langle 10|\hat{U}^2|00\rangle, \tag{10}$$

where $c_{10}^{(2)}$ depends on n and m in the state $|\tilde{1}\rangle$. By writing (10), we used the well-known property of completeness

$$\sum_{\tilde{m}} |\tilde{m}\rangle\langle \tilde{m}| = 1, \tag{11}$$

where the sum is taken over all states of both atoms. By considering only the terms with c_{00}, c_{10}, and c_{01} in (2), relation (4) yields the first part of the DM of atom A:

$$\mathbf{d}_1^A = \int d\mathbf{r}_1^A d\mathbf{r}_2^A d\mathbf{r}_1^B d\mathbf{r}_2^B \hat{\mathbf{d}}^A |c_{00}\Psi_0^A\Psi_0^B + \sum_{\tilde{1}}[c_{10}^{(2)}\Psi_1^A\Psi_0^B + c_{01}^{(2)}\Psi_0^A\Psi_1^B]|^2 \tag{12}$$

$$\approx c_{00}^* \sum_{\tilde{1}} c_{10}^{(2)}\langle 00|\hat{\mathbf{d}}^A|10\rangle + \text{c.c.} = \frac{c_{00}^*}{2\triangle E^2} \sum_{\tilde{1}} \langle 00|\hat{\mathbf{d}}^A|10\rangle\langle 10|\hat{U}^2|00\rangle + \text{c.c.}$$

Here, $\sum_{\tilde{1}}$ is the sum over all states $|\tilde{1}\rangle$, i.e., over $m = 0; \pm 1$ and $n = 2, 3, \ldots, \infty$. Besides $|10\rangle$, we include all the remaining states $|\tilde{m}_A\tilde{m}_B\rangle$ in (12): $|1^A\rangle = |1snp\rangle$, and also $|1sns\rangle$, $|1snd\rangle$, $|1snf\rangle$, etc.; and, in addition to $|0^B\rangle$, all higher states $|\tilde{m}\rangle$. This is possible to make because $\langle 00|\hat{\mathbf{d}}^A|\tilde{m}_A\tilde{m}_B\rangle = 0$ for them. Then, with regard for (11), we obtain

$$\mathbf{d}_1^A \approx \frac{c_{00}^*}{2\triangle E^2} \sum_{\tilde{m}_A\tilde{m}_B} \langle 00|\hat{\mathbf{d}}^A|\tilde{m}_A\tilde{m}_B\rangle\langle \tilde{m}_A\tilde{m}_B|\hat{U}^2|00\rangle + \text{c.c.} = \frac{c_{00}}{\triangle E^2}\langle 00|\hat{\mathbf{d}}^A\hat{U}^2|00\rangle. \tag{13}$$

Relation (11) and the normalization conditions for Ψ_0^{AB} yield

$$c_{00}^2 = 1 - \sum_{l_1 j_1 \neq (0,0)} c_{l_1 j_1}^2 \approx 1 - \frac{\langle 00|\hat{U}^2|00\rangle}{4\triangle E^2} = 1 + \frac{E_{VdV}}{2\triangle E}, \tag{14}$$

where $E_{VdV} \approx -\frac{\langle 00|\hat{U}^2|00\rangle}{2\triangle E} \approx -2.15e^2(a_B^5/R^6)$ is the van der Waals interaction energy for two atoms He^4. We have $|E_{VdV}| \ll \triangle E$; therefore, $c_{00} \approx 1$.

With regard for (2) and (4), the total DM of atom A is given by the formula

$$\mathbf{d}^A = \sum_{\tilde{m}_1^A\tilde{m}_2^A\tilde{m}_1^B\tilde{m}_2^B} c_{\tilde{m}_1^A\tilde{m}_1^B}^* c_{\tilde{m}_2^A\tilde{m}_2^B} \langle \tilde{m}_1^A\tilde{m}_1^B|\hat{\mathbf{d}}^A|\tilde{m}_2^A\tilde{m}_2^B\rangle. \tag{15}$$

The above-derived expression for $\mathbf{d}_1$ is a part of the last relation from the terms $c_{00}c_{10}$ in the second order of PT. The formula for $\mathbf{d}^A$ includes also the nonzero contribution in the first order of PT from terms of the form $c_{11}^{(1)}c_{21}^{(1)}$, $c_{22}^{(1)}c_{21}^{(1)}$, and higher-order ones which behave themselves, respectively, as $\sim (a_B/R)^7$, $\sim (a_B/R)^9$, etc. (see below). At l and j which are not equal to zero $(\tilde{0})$ simultaneously, the relation

$$c_{lj}^{(1)} = \frac{\langle l^A j^B|\hat{U}|0^A 0^B\rangle}{E_0^{(0)} - E_j^{(0)} + E_0^{(0)} - E_l^{(0)}} \tag{16}$$

is valid. Since $\langle lj|\hat{U}|00\rangle \neq 0$ only at $l, j \geq 1$, we set $2E_0^{(0)} - E_j^{(0)} - E_l^{(0)} \approx -2\triangle E$. Since the contribution in the DM from c_{00} is taken into account in $\mathbf{d}_1$, we redefine c_{00} in (15) according to (16) with $-2\triangle E$ in the denominator (such c_{00} is equal to zero). Since $c_{10}^{(1)} = 0$, we will write also $-2\triangle E$ in the denominator for it instead of $-\triangle E$. We can now use the relation (11), and relation (15) minus $\mathbf{d}_1$ is reduced to

$$
\begin{aligned}
\mathbf{d}_2^A &\approx \frac{1}{4\triangle E^2} \sum_{j_1 l_1 j_2 l_2} \langle 00|\hat{U}|l_1 j_1\rangle \langle l_1 j_1|\hat{\mathbf{d}}^A|l_2 j_2\rangle \langle l_2 j_2|\hat{U}|00\rangle = \\
&= \frac{1}{4\triangle E^2} \langle 0^A 0^B|\hat{\mathbf{d}}^A \hat{U}^2|0^A 0^B\rangle = \frac{\mathbf{d}_1^A}{4}.
\end{aligned} \tag{17}
$$

Whence we obtain the total DM of atom A in the principal $(\sim (a_B/R)^7)$ approximation:

$$
\mathbf{d}^A = \mathbf{d}_1^A + \mathbf{d}_2^A \approx \frac{5}{4\triangle E^2} \langle 0^A 0^B|\hat{\mathbf{d}}^A \hat{U}^2|0^A 0^B\rangle. \tag{18}
$$

Using the formulas for the operator of charge $\hat{Q}$ [10], we represent the interatomic potential $\hat{U}$ as the sum

$$
\hat{U} = \hat{Q}_A^+ \hat{Q}_B \frac{1}{R} = \hat{U}_{dd} + \hat{U}_{qd} + \hat{U}_{qq} + \dots, \tag{19}
$$

where

$$
\hat{U}_{dd} = \frac{\hat{\mathbf{d}}^A \hat{\mathbf{d}}^B}{R^3} - \frac{3(\hat{\mathbf{d}}^A \mathbf{R})(\hat{\mathbf{d}}^B \mathbf{R})}{R^5} \tag{20}
$$

is the dipole-dipole part of the potential, and

$$
\begin{aligned}
\hat{U}_{qd} &= \frac{3}{R^4} \sum_{\alpha\beta=1}^{3} \left(\hat{Q}_{\alpha\beta}^A \left[\hat{d}_\beta^B n_\alpha + \hat{d}_\alpha^B n_\beta - 5\hat{\mathbf{d}}^B \mathbf{n} n_\alpha n_\beta \right] \right) - (A \leftrightarrow B) \\
&+ \frac{3}{R^4} \hat{\mathbf{d}}^B \mathbf{n} \sum_{\alpha=1}^{3} \hat{Q}_{\alpha\alpha}^A - (A \leftrightarrow B)
\end{aligned} \tag{21}
$$

is the dipole-quadrupole part. The quadrupole-quadrupole part, $\hat{U}_{qq}$, includes a negligible correction to the DM $(\sim (a_B/R)^9$ from $\hat{U}_{qd}\hat{U}_{qq})$. Therefore,

$$
\hat{U}^2 \approx \hat{U}_{dd}^2 + 2\hat{U}_{dd}\hat{U}_{qd} + \hat{U}_{qd}^2. \tag{22}
$$

In view of the parity relative to the inversion, we preserve only the term $2\hat{U}_{dd}\hat{U}_{qd}$. As a result, we obtain

$$
\mathbf{d}^A \approx \frac{5}{2\triangle E^2} \langle 0^A 0^B|\hat{\mathbf{d}}^A \hat{U}_{dd}\hat{U}_{qd}|0^A 0^B\rangle. \tag{23}
$$

Let us choose the Z-axis along $\mathbf{R}$: $\mathbf{R} = R\mathbf{i}_z$. Then a nonzero contribution to $\mathbf{d}^A$ is given by the terms

$$
\hat{\mathbf{d}}^A \hat{U}_{dd}\hat{U}_{qd} = \frac{6}{R^7} \left\{ (\hat{d}_x^B)^2 \hat{d}_x^A \hat{d}_z^A \hat{Q}_{xz}^A + (x \leftrightarrow y) - (\hat{d}_z^B)^2 (\hat{d}_z^A)^2 (\hat{Q}_{xx}^A + \hat{Q}_{yy}^A - 2\hat{Q}_{zz}^A) \right\}. \tag{24}
$$

With regard for the isotropy $\Psi_0 = |0\rangle$, we eventually arrive at the general formula

$$
\mathbf{d}^A \approx \frac{30\mathbf{i}_z}{R^7 \triangle E^2} \langle 0^B|(\hat{d}_z^B)^2|0^B\rangle \left(\langle 0^A|\hat{d}_x^A \hat{d}_z^A \hat{Q}_{xz}^A|0^A\rangle + \langle 0^A|(\hat{d}_z^A)^2(\hat{Q}_{zz}^A - \hat{Q}_{xx}^A)|0^A\rangle \right), \tag{25}
$$

which can be rewritten in the following simple form:

$$
\mathbf{d}_A = -D_7 |e| \frac{a_B^8}{R^7} \mathbf{n}, \tag{26}
$$

$$D_7 = Z_A Z_B \left(\frac{2Ry}{\triangle E} \right)^2 \langle \tilde{r}_1^2 \rangle_B \langle \tilde{r}_1^4 \rangle_A. \tag{27}$$

Here, $\langle \tilde{r}_1^2 \rangle_B \equiv \langle 0^B | (r_1^B / a_B)^2 | 0^B \rangle$, $\langle \tilde{r}_1^4 \rangle_A \equiv \langle 0^A | (r_1^A / a_B)^4 | 0^A \rangle$, $Ry = e^2 / 2a_B = 13.6\, eV$.

The next correction to $\mathbf{d}_A$ is of the order of $\sim (a_B/R)^9$, and it is $\sim (R/a_B)^2$ times less than (26). For He II, $R \sim 3.6\,\text{Å}$, $(R/a_B)^2 \simeq 50$; therefore, it can be neglected. It is of interest that the separate corrections to D_7 for various n and m in the $(\tilde{1}\tilde{1})$ and $(\tilde{1}\tilde{2})$ states from (2) are small, about 0.1 and less. Moreover, we have an alternating series for $\mathbf{d}_A$, so that several hundreds of various corrections and about 10 different n significantly contribute to the total DM D_7 (27).

The positive sign of D_7 means that the electron cloud of each of two atoms He^4 shifts to the neighboring atom. That is, the interatomic space acquires an excess of the negative charge.

To calculate the value of D_7, we need to know the wave function $|0\rangle$ of the ground state of an atom He^4. The exact analytic solution is not available for it, but some approximations have been proposed. Within the simplest one-parameter one [17, 19]

$$\Psi_0 = \varphi_{1s}(\mathbf{r}_1) \varphi_{1s}(\mathbf{r}_2), \ \varphi_{1s}(\mathbf{r}) = \sqrt{Z^{*3} / \pi a_B^3} e^{-rZ^*/a_B} \tag{28}$$

$(Z^* = Z - 5/16)$, we get $\langle \tilde{r}_1^2 \rangle = 3/(Z^*)^2 \approx 1.053$, $\langle \tilde{r}_1^4 \rangle = 45/2(Z^*)^4 \approx 2.775$. In (27) for $\triangle E$, we take the least $(\triangle E_1(n=2))$ and largest $(\triangle E_{ion})$ values, so we obtain $D_7 \approx 14.32 \div 19.18$ (here we take into account also that $Z_A = Z_B = 2$).

The more exact multiparameter Ψ_0 were calculated within various methods. In [20], it was proposed to seek Ψ_0 in the form

$$\Psi_0(s,t,u) = \varphi(ks, kt, ku), \quad \varphi(s,t,u) = A e^{-s/2} \sum_{l,n,m=0}^{\infty} c_{n,2l,m} s^n t^{2l} u^m, \tag{29}$$

where

$$s = r_1 + r_2, \ t = r_1 - r_2, \ u = r_{12}. \tag{30}$$

The formalism in the variables s, u, t is given in [21] in detail. For the 3-parameter Ψ_0 [20, 21],

$$\varphi = A e^{-s/2}(1 + c_1 u + c_2 t^2), \quad c_1 \approx 0.081, \ c_2 \approx 0.01, \ k \approx 3.63, \tag{31}$$

the calculation yields

$$A = (k^3/4\pi)(4 + 35c_1 + 48c_2 + 96c_1^2 + 576c_2^2 + 308c_1 c_2)^{-1/2} \approx 1.325, \tag{32}$$

$$\langle \tilde{r}_1^2 \rangle = \frac{768\pi^2 A^2}{k^8} \left(1 + \frac{189}{16}c_1 + 24c_2 + 42c_1^2 + 480c_2^2 + 195c_1 c_2 \right), \tag{33}$$

$$\langle \tilde{r}_1^4 \rangle = \frac{23040\pi^2 A^2}{k^{10}} \left(1 + \frac{979}{64}c_1 + 52c_2 + 68c_1^2 + 1704c_2^2 + \frac{8305}{16}c_1 c_2 \right). \tag{34}$$

We have also performed the numerical calculations for 6-, 10-, and 20-parameter Hylleraas functions [20, 22, 23] and for more general 10-, 39-, and 80-parameter Kinoshita functions [24, 25] of the form

$$\Psi_0(s,t,u) = \varphi(ks, kt, ku), \quad \varphi(s,t,u) = A e^{-s/2} \sum_{l,n,m=0}^{\infty} c_{n,m,l} s^{n+1} (t/u)^{2l} (u/s)^m \tag{35}$$

which include the negative degrees in the expansion. The results are given in the Table. For the 3-parameter Ψ_0, the result is obtained numerically and analytically. It is seen from the

Number of parameters of Ψ_0	Ref.	$\langle \tilde{r}_1^2 \rangle$	$\langle \tilde{r}_1^4 \rangle$	D_7^{min}	D_7^{max}	$\bar{D}_7$	$\left| \frac{J^{theor} - J^{exp}}{J^{exp}} \right|$
1	[19, 17]	1.053	2.775	14.32	19.18	15.53	$5 \cdot 10^{-2}$
3	[20, 19]	1.182	3.815	22.1	29.6	23.98	$2.5 \cdot 10^{-3}$
6	[20, 19]	1.188	3.861	22.47	30.1	24.38	$5 \cdot 10^{-4}$
10	[22]	1.249	4.222	25.82	34.6	28.02	$1 \cdot 10^{-4}$
10	[24]	1.19	3.91	22.79	30.54	24.73	$1 \cdot 10^{-6}$
20	[23]	1.19326	3.967	23.19	31.07	25.16	$2 \cdot 10^{-6}$
39	[25]	1.19314	3.961	23.15	31.02	25.12	$1 \cdot 10^{-6}$
80	[25]	1.19346	3.973	23.23	31.12	25.2	$2 \cdot 10^{-7}$
1078	[26]	1.19348	–	–	–	–	$5 \cdot 10^{-7}$

Table that the more the number of parameters, the less the variation of the results for $\langle \tilde{r}_1^2 \rangle$, $\langle \tilde{r}_1^4 \rangle$, and D_7. Moreover, the convergence is quite good. For comparison, we present also a deviation of the theoretical ionization potential J of atoms He^4 from the experimental value, $J^{exp} \approx 198310.8 \, cm^{-1}$, by the data from [19, 25, 26]. It is seen that the convergence for the energy is faster, which is natural. Indeed, at the determination of Ψ_0, namely the energy is minimized. However, the quantities $\langle \tilde{r}_1^2 \rangle$ and $\langle \tilde{r}_1^4 \rangle$ are determined also with a high accuracy. In work [26], Ψ_0 was calculated by means of the expansion in Laguerre polynomials. Up to 1078 terms were taken into account, and the determined value of $\langle \tilde{r}_1^2 \rangle$ (see Table) coincides with accuracy to 4 decimal points with the value obtained by us for the 80-parameter Ψ_0. This means that the above-determined $\langle \tilde{r}_1^2 \rangle$, $\langle \tilde{r}_1^4 \rangle$, and the upper and lower limits of D_7 for this Ψ_0 have error less than 0.1%. The modern more exact methods of calculations of Ψ_0 are available (see, e.g., [27, 28]), but the too high accuracy is not required for our task, because it will not increase the accuracy of D_7, since the large error of D_7 remains unremovable due to the "smearing" of $\triangle E$.

Since the main contribution to the DM is given by the states with large n, for which $\triangle E \approx \triangle E_{ion}$, D_7 should be closer to the lower limit. Therefore, we define D_7 in the table as

$$\bar{D}_7 \approx D_7^{min} + (D_7^{max} - D_7^{min})/4. \tag{36}$$

In view of the result for the 80-parameter Ψ_0, we finally have

$$D_7 \approx \bar{D}_7 \approx 25.2 \pm 2. \tag{37}$$

Solution (26), (37) is in agreement with (1), though we have obtained a somewhat larger value for D_7. We note that the formula different from (27) for D_7 was derived in [15, 16] with the use of dispersion sums. The most exact value, $D_7 = 18.4$, was obtained there on the basis of the 20-parameter Ψ_0 taken from [23]. In this case, certain approximations were used in calculations, but the error arising due to these approximations was not evaluated. For this and more exact functions Ψ_0, we have calculated a more larger value of D_7 (37). The difference of values of D_7 in (37) and (1) is probably related to the approximations made in both calculations. We note that the above-proposed calculation is much simpler than that in [15, 16].

3 Conclusions

The above-considered (and earlier in [15, 16]) tidal mechanism of the polarization of atoms is, in our opinion, a key to the comprehension of electric properties of superfluid He^4 and other fluids of nonpolar atoms, though we cannot yet exclude the contribution of other mechanisms (for example, the gradient thermoemf arising in an electrode due to the gradient of temperature). At the present time, the modeling of the near-surface polarization of helium

near the electrode seems to be the most promising for the explanation of observed electric signals [1, 2]. And we do not exclude the possibility that the researchers of this field will meet a number of surprises.

[1] A.S. Rybalko, *Fiz. Nizk. Temp.* **30**, 1321 (2004) [*Low Temp. Phys.* **30**, 994 (2004)].

[2] A.S. Rybalko and S.P. Rubets, *Fiz. Nizk. Temp.* **31**, 820 (2005) [*Low Temp. Phys.* **31**, 623 (2005)].

[3] A. Rybalko, S. Rubets, E. Rudavskii, V. Tikhiy, S. Tarapov, R. Golovashchenko and V. Derkach, *Phys. Rev.* **B 76**, 140503 (2007).

[4] A.M. Kosevich, *Low Temp. Phys.* **31**, 37, 920 (2005).

[5] L.A. Melnikovsky, *J. Low Temp. Phys.*, **148**, 559 (2007).

[6] E.A. Pashitskii and S.M. Ryabchenko, *Low Temp. Phys.* **33**, 8 (2007).

[7] K.V. Grigorishin and B.I. Lev, *Ukr. J. Phys.* **53**, 645 (2008).

[8] M.D. Tomchenko, submitted in *J. Low Temp. Phys.*, arXiv:cond-mat/0903.4538 v2 (2009).

[9] G.E. Volovik, *Pis'ma Zh. Eksp. Teor. Fiz.* **39**, 169 (1984) [*JETP Lett.* **39**, 200 (1984)].

[10] V.M. Loktev and M.D. Tomchenko, *Fiz. Nizk. Temp.* **34**, 337 (2008) [*Low Temp. Phys.* **34**, 262 (2008)].

[11] V.D. Natsik, *Fiz. Nizk. Temp.* **34**, 625 (2008).

[12] E.D. Gutlyanskii, *Fiz. Nizk. Temp.* **35**, 956 (2009).

[13] S.I. Shevchenko and A.S. Rukin, *Pis'ma Zh. Eksp. Teor. Fiz.* **90**, 46 (2009).

[14] M.D. Tomchenko, "Possible explanation of the electric activity of He-II in experiments with second sound", submitted to *Fiz. Nizk. Temp.*

[15] W. Byers Brown and D.M. Whisnant, *Mol. Phys.* **25**, 1385 (1973).

[16] D.M. Whisnant and W. Byers Brown, *Mol. Phys.* **26**, 1105 (1973).

[17] I.O. Vakarchuk, *Quantum Mechanics*, L'viv National University, L'viv (2004).

[18] G. Herzberg, *Atomic Spectra and Atomic Structure*, [Russian translation] IL, Moscow (1948), Chap. 1, 6.

[19] H.A. Bethe and E.Ė. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms*, Springer, Berlin (1957), Chap. 2.

[20] E. Hylleraas, *Z. Phys.* **48**, 469 (1928), **54**, 347 (1929).

[21] A. Sommerfeld, *Atombau und spektrallinien*, vol. 2, Friedr. Vieweg und Sohn, Braunschweig (1951), Chap. 10.

[22] S. Chandrasekhar, D. Elbert and G. Herzberg, *Phys. Rev.* **91**, 1172 (1953).

[23] J. F. Hart and G. Herzberg, *Phys. Rev.* **106**, 79 (1957).

[24] T. Kinoshita, *Phys. Rev.* **105**, 1490 (1957).

[25] T. Kinoshita, *Phys. Rev.* **115**, 366 (1959).

[26] C. L. Pekeris, *Phys. Rev.* **115**, 1216 (1959).

[27] S. P. Goldman, *Phys. Rev. Lett.* **73**, 2547 (1994).

[28] J. Mitroy, M. W. J. Bromley and K. Ratnavelu, arXiv:physics/0607232.